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TURBULENT ENERGY GENERATED BY ACCELERATIONS AND SHOCKS

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ABSTRACT

We calculate the turbulent energy generated at the interface between two fluids undergoing a constant acceleration or a shock . Assuming linear density profiles in the mixed region we find $E_{turbulent}/E_{directed} = 2.3A^2$ % (constant acceleration) and $9.3A^2$ % (shock) , where A is the Atwood number . Diffusion models predict somewhat less turbulent energy and a density profile with a tail extending into the lower density fluid . Eddy sizes are approximately 27% (constant acceleration) and 17% (shock) of the mixing depth into the heavier fluid .

A common phenomenon is the mixing of two fluids in an unstable configuration , i.e. , a light fluid supporting a heavier fluid . The mixing continues until the two fluids have interchanged their positions . At the end of the process it is straightforward to calculate the turbulent energy , which must have dissipated into heat , by taking the difference in potential energy between the initial and final configurations . The turbulent energy **during** the process of mixing , however , is not so easily calculated or measured .

During the process of mixing the potential energy can still be calculated or measured if we know how much mass has "fallen" by that time . This calls for the density profile $\rho(y)$ between the two original fluids of density $\rho_{\pm}$:

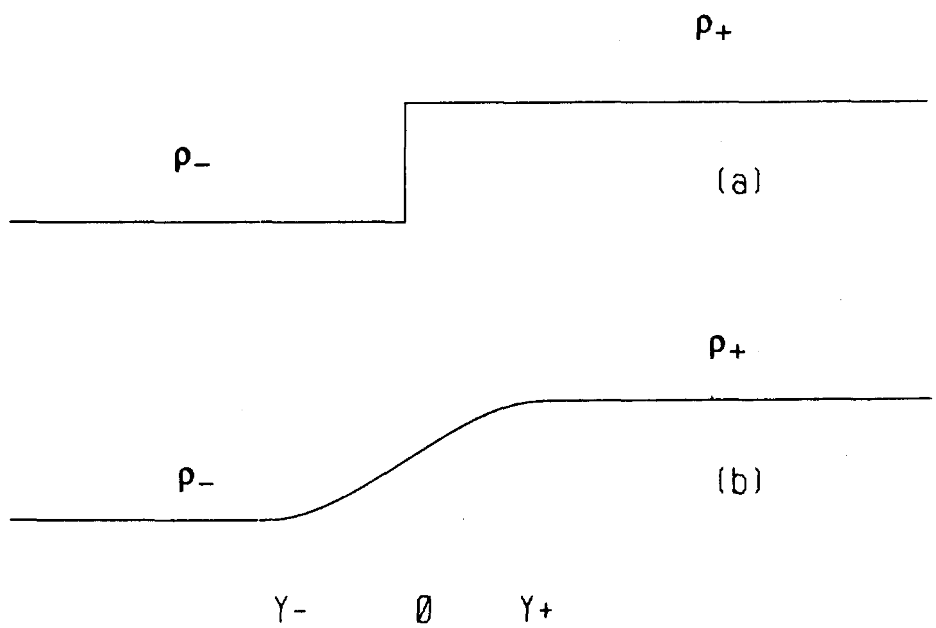


Fig. 1

Neglecting dissipation , the turbulent energy $E_{turbulent}$ can be equated to the difference in potential energy between the unmixed (Fig. 1a) and the mixed (Fig 1b) density profiles :

$$E_{turbulent} = g(\rho_+ y_+^2 - \rho_- y_-^2)/2 - g\int_{y_-}^{y_+} \rho(y) y dy \qquad (1)$$

where g = acceleration and $y_\pm$ are the extent of mix into the heavy and light fluids of density $\rho_\pm$ respectively .

If we assume that $\rho(y)$ is linear between $y_\pm = \pm h$, then the first term in Eq.(1) gives $(g/2)[\rho_+ - \rho_-]h^2$, while the second term gives $(g/3)[\rho_+ - \rho_-]h^2$, hence

$$E_{turbulent}(\text{linear profile}) = (g/6)[\rho_+ - \rho_-]h^2 \quad . \qquad (2)$$

We define

$$E_{directed} = (v^2/2)\int \rho(y) dy = \frac{1}{2}(gt)^2 [\rho_+ y_+ - \rho_- y_-] \qquad (3)$$

where the second equality follows from conservation of mass and is true for any density profile . For the linear density profile :

$$E_{directed}(\text{linear profile}) = (1/2)(gt)^2[\rho_+ + \rho_-]h \qquad (4)$$

so that

$$\frac{E_{turbulent}}{E_{directed}}(\text{linear profile}) = \frac{1}{3}\frac{hA}{gt^2} \qquad (5)$$

where A = Atwood number = $[\rho_+ - \rho_-]/[\rho_+ + \rho_-]$.

To determine h we use the recent experimental results of Read and Youngs[1] who find

$$y_+ = h = 0.07Agt^2 \quad ; \qquad (6)$$

hence

$$\frac{E_{turbulent}}{E_{directed}} \text{(linear profile)} = \frac{0.07}{3} A^2 \quad , \qquad (7)$$

implying that the ratio of turbulent to directed energy is at most 2.3 % .

The experimentally measured density profiles are not quite linear and indicate that $|y_-| > |y_+|$, i.e. the mixing extends deeper into the light fluid than into the heavy fluid . A model that gives such a density profile was developed several years ago by Belenkii and Fradkin[2] and it was shown by C. Leith[3] that it is in agreement with the experiments of Read and Youngs . The Belenkii and Fradkin model is a turbulent diffusion model which assumes that the density profiles are given by a similarity solution to the diffusion equation . In a recent study[4] we point out that the model is essentially a one-parameter model , and construct a similar model for shocks . That parameter can be taken to be λ/y_+ , where λ is a measure of eddy size .

The model for shocks is constructed by letting $g \rightarrow \Delta v\, \delta(t)$ as was done by Richtmyer[5] for single-scale perturbations in the linear regime. Earlier experiments by Meshkov[6] lent support for Richtmyer's approach . Experiments are now underway at CalTech where both the linear and turbulent regimes will be studied in a shock tube .

With the understanding that our diffusion model for shocks has yet to be checked against experiments , it is interesting to compare accelerations with shocks :

$$y_+ = 0.07 \ Agt^2 \qquad \text{(constant acceleration)} \qquad (8a)$$

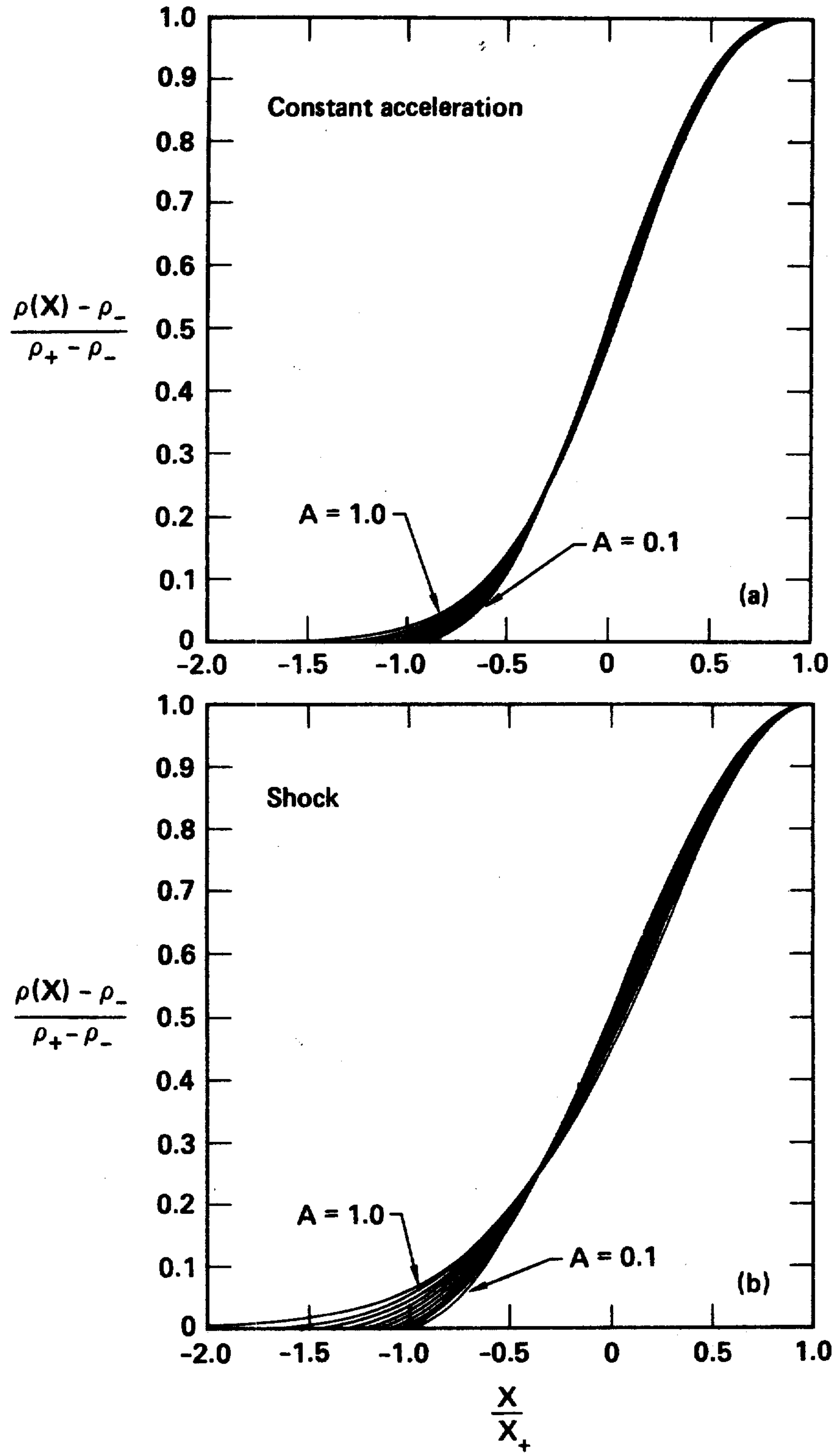
$$y_+ = 0.14 \ A\Delta vt \qquad \text{(shock)} \qquad (8b)$$

$$26\% \leq \lambda/y_+ \leq 29\% \qquad \text{(constant acceleration)} \qquad (9a)$$

$$16\% \leq \lambda/y_+ \leq 18\% \quad . \qquad \text{(shock)} \qquad (9b)$$

The range of λ/y_+ correlates with the range of the Atwood number ($0.1 \leq A \leq 1.0$) in each case . Eqs. (9a) and (9b) are obtained by requiring that the mixing depth y_+ into the heavier fluid in the turbulent diffusion models for accelerations and for shocks agree with Eqs. (8a) and (8b) respectively .We see that eddy sizes , measured relative to their respective mixing depths , do not vary much as a function of A ; they are , however , predicted to be smaller for shocks suggesting that mixing generated by constant accelerations and by shocks may be characterized as "chunk mix" and "atomic mix" respectively (for details see Ref. 4) .

The density profiles calculated in the turbulent diffusion models are shown in Fig. 2 (X is a dimensionless parameter related to y via gt^2 and Δvt for a constant acceleration and a shock respectively) :



Constant acceleration
A = 1.0
A = 0.1
(a)
Shock
A = 1.0
A = 0.1
(b)
$\frac{\rho(X) - \rho_-}{\rho_+ - \rho_-}$
$\frac{X}{X_+}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-2.0
-1.5
-1.0
-0.5
0
0.5
1.0

Fig. 2

Referring to Fig. 2 , we see that in both cases the mixing depth into the lighter fluid $|y_-|$ is larger than the mixing depth into the heavier fluid $|y_+|$, but it is even more so for shocks : the asymmetry $|y_- / y_+|$ for A = 1 is about 2 for a constant acceleration , but it is about 2.8 for a shock .

Using the above density profiles we obtain (numerically) the ratio of turbulent to directed energy as a function of Atwood number A . These are shown in Fig. 3 :

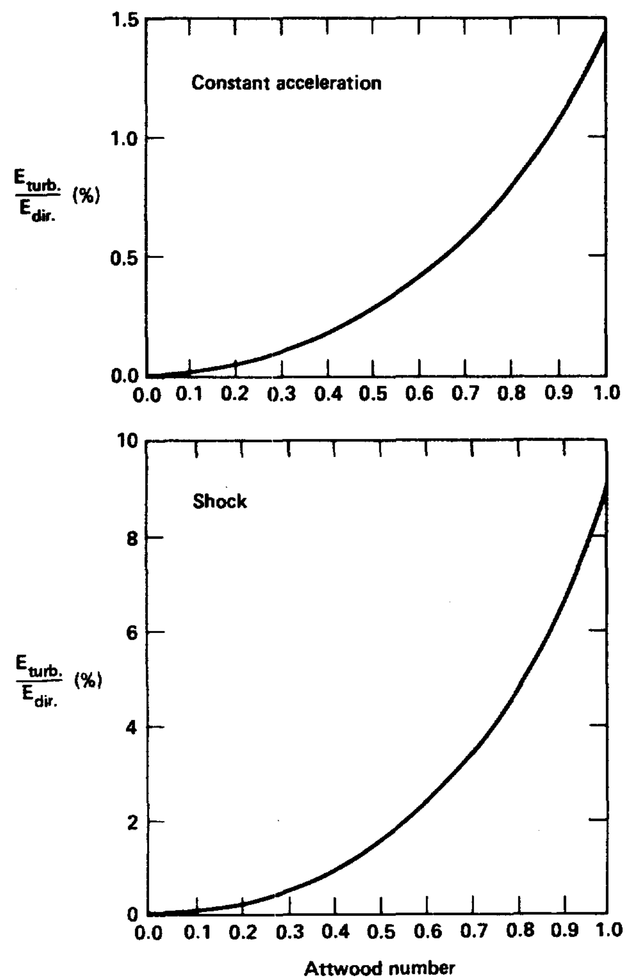


Fig. 3

It is interesting to compare Fig. 3 with the results that we obtain using the simple linear density profile as discussed at the beginning of this paper :

$$\frac{E_{turbulent}}{E_{directed}} \text{ (linear profile)} = \frac{0.07}{3} A^2 \qquad \text{(constant acceleration)} \qquad (10a)$$

$$\frac{E_{turbulent}}{E_{directed}} \text{ (linear profile)} = \frac{0.28}{3} A^2 \qquad \text{(shock)} \qquad (10b)$$

The factor of 4 comes from treating a shock as an instantaneous acceleration , and is of course included in our turbulent diffusion model for shocks : the only difference between Figs. 3a and 3b on the one hand and Eqs. (10a) and (10b) on the other is the density profile used to calculate $E_{turbulent}$. We see from Figs. 3a and 3b that the ratio of $E_{turbulent}$ to $E_{directed}$ is almost quadratic in A , and has a maximum of 1.4% and 9.3% for a constant acceleration and a shock respectively , in fair agreement with the linear density results Eqs. (10a) and (10b) which give an exactly quadratic function of A and upper maxima of 2.3% and 9.3% for a constant acceleration and a shock respectively .

It is only a coincidence that the linear density profile and the turbulent diffusion density profile give the same maximum value 9.3% for the ratio $E_{turbulent} / E_{directed}$ in the case of a shock . But it is true that in general this ratio is not too sensitive to the precise shape of the density profile essentially because we integrate over it .

Details of the above calculations and what they imply about Inertial Confinement Fusion capsules are given in Ref. 4 .

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